



# Institute of Mathematical Sciences

MAGNETO-FLUID DYNAMICS DIVISION

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SHERWOOD PROGRESS REPORT

No. 5

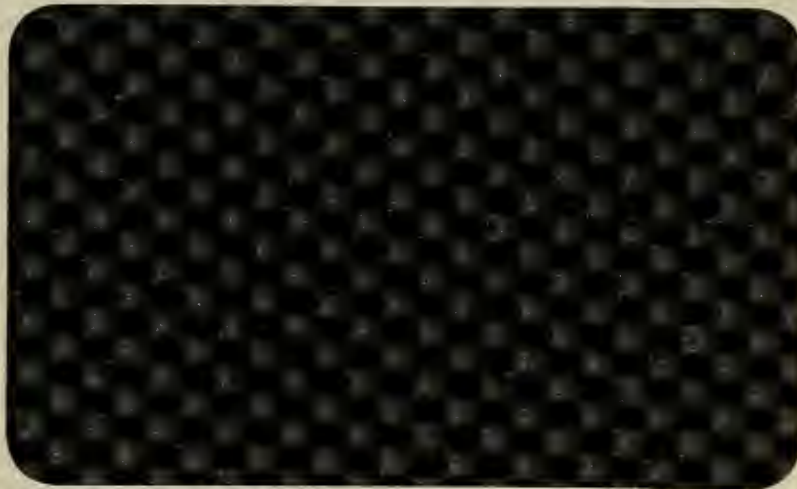
(January 1961-December 1961)

J. Berkowitz

April 16, 1962

NEW YORK UNIVERSITY

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## Introduction

This report is a summary of activities related to Project Sherwood at the Magneto-Fluid Dynamics Division of the Courant Institute of Mathematical Sciences of New York University from January 1961 to December 1961. Substantial investigations are summarized under the following subject headings:

- Charged Particle Trajectories
- Cusped Geometries
- Collisionless Shock Theory
- Wave Propagation
- Other Subjects.

There is also a brief summary of personnel information and a bibliography of reports and papers published in the period covered. For a complete earlier bibliography see Sherwood Progress Reports No. 3, NYO-2544 (1959), and No. 4, NYO-9394 (1961).



## Personnel

The average number of equivalent full-time scientific staff members was approximately 16, accounted for by approximately 20 individuals who spent an appreciable amount of time on this project. Approximately 11 man-years were contributed by senior staff members (Ph.D. level). In addition there were 3 assistant research scientists, 2 graduate students, 1 active consultant, and approximately 2 clerical and administrative persons.

## Charged Particle Trajectories

### Adiabatic:

The adiabatic motion of the guiding center of a single charged particle in an electromagnetic field has previously been analyzed by Kruskal and others as an asymptotic series expansion of the true trajectory, cf. Berkowitz and Gardner [Ref. 1, PR 4]. A simple penetrating treatment of the problem, using perturbation methods of classical analysis, was given by Gardner [Ref. 3, PR 4], yielding a very easy demonstration of the invariance of the magnetic moment to all orders in the expansion parameter. Taniuti [13] has given an invariant treatment of the canonical theory of the motion, exhibiting explicitly the Hamiltonian in terms of the coordinates of gyration and the drift.

### Non-adiabatic:

The underlying assumption which is required for the validity of the adiabatic theory is that in one Larmor gyration the particle experiences only a relatively small change in the electromagnetic field. This is certainly not the case in the neighborhood of a point where the magnetic field is zero. General estimates of non-adiabatic effects are not available, nor are they to be expected, although in some fortuitous situations some special results have been obtained (by Hartweck and Schlüter, and by Chandrasekhar). A detailed numerical study has been made of particle orbits in a magnetic field of



the cusped geometry type; see van Norton [17], and Grad and van Norton [21]. Two features which have emerged from the computations are that there is a narrow transition zone between very adiabatic and very non-adiabatic orbits, and that even the slightly non-adiabatic orbits are so complex that their description is amenable to statistical methods. Satisfactory containment is possible even in the absence of an adiabatic invariant. The results do not seem strongly dependent on the particular magnetic field configuration, and may therefore be applied to other problems, e.g., mirror machine containment.

A feature of this class of slightly non-adiabatic orbits that may yield to mathematical analysis is the following phenomenon, observed from the numerical experiments: The guiding center remains on a line of force and the magnetic moment remains almost constant, except at the point where the guiding center is closest to the zero point of the magnetic field; here the magnetic moment jumps to another slightly different almost constant value. This slight change in magnetic moment in turn produces a change of the exact point at which the particle is reflected at the mirror and this in turn produces a large difference in phase. Thus two particles initially in the same state except for slightly different phases will, by the second time they pass near the zero point of the field, have vastly different phases. Thus one can consider the successive increments in magnetic moment as

essentially uncorrelated with the original phase of the particle. This randomization of phase makes possible a stochastic treatment of this non-adiabatic effect.

Based on some profound mathematical discoveries of Moser [33a], Gardner has recently shown that, essentially, if the magnetic field is strong enough a particle not in the adiabatic loss cone is contained for all time. This result is not the outcome of approximate asymptotic analysis, but stems from a rigorous mathematical theorem which reveals the complicated structure of the manifold of charged particle orbits.

Berkowitz and Gardner [Ref. 1, PR 4]

Gardner [Ref. 3, PR 4]

Taniuti [13]

van Norton [17]

Grad and van Norton [21]

Moser [33a]

## Cusped Geometries

Stimulated in part by the increased experimental activity in cusped geometry, there has been some effort to review and improve the available theory.

Grad [12] presented a descriptive survey of the current theoretical picture of plasma containment in cusped magnetic configurations, together with some indications and cautions about the relevance of the theory to points of contact with experiments.

Considerable effort was made to refine the containment theory and general results were obtained for cusped plasma systems in the situation with arbitrary  $\beta$  where plasma and field are mixed together. A numerical study of single particle orbits in cusped magnetic fields (van Norton [17]) was used to test the conjecture (Grad [Ref. 22, PR 4]) that the particles can be classified into three types: (1) those whose motion is adiabatic, like particles contained in a mirror machine, (2) those whose motion is wildly non-adiabatic, like those in an idealized  $\beta = 1$  sharp interface cusped plasma, and finally (3) a relatively thin transition zone of slightly non-adiabatic particles. Thus, to obtain estimates for containment times, one combines the mirror-type loss rates for the adiabatic particles, and cusp-type loss rates for the non-adiabatic ones. The resulting formulas are given by Grad [12], together with modifications caused by the presence of electric fields. Since there is different scaling with regard to both temperature and

density in the two types of loss mechanism, considerable care must be exercised to interpret containment results. Moreover, the loss rates can be very sensitive to electrostatic potentials which may well be altered by slight changes in experimental arrangements.

In developing a containment theory for cusped geometries, as well as for many other problems in plasma physics, it is important to have a theory of the structure of the steady transition layer separating a uniform vacuum magnetic field from a collisionless field-free plasma. A solution for the thinnest transition layer, corresponding to any non-singular velocity distribution prescribed deep in the plasma, provided there are no trapped particles, was given by Grad [Ref. 25, PR 4]. A striking feature of the solution is that the magnetic field penetrates only a finite distance into the plasma, whereas the plasma decays exponentially into the magnetic field region. The scale of length is the Larmor radius. This solution was obtained ignoring the electric fields arising from charge separation in the case of particles of equal masses. Numerical computations are underway (Weitzner, Paskievici, Sestero) to take these electric fields into account, as well as to study the effect of trapped particles on the transition layer.

The thinnest boundary layer is desirable for stability and low particle losses for cusped geometries, and it is best as well to minimize the cyclotron radiation. Using the sheath analysis just mentioned, the minimum radiation has been

determined by Burkhardt [22],

$$W = 1 \times 10^{-32} T_o^2 n_o^{3/2} ,$$

where  $W$  is in watts/cm.<sup>2</sup>,  $T_o$  is in electron volts, and  $n_o$  is the number of electrons per cm.<sup>3</sup>. This amount of radiation turns out to be tolerable for plasmas of thermo-nuclear interest, but this is the minimum that can be achieved and not necessarily a good estimate.

Some mathematical analysis is being carried out to determine details of cusp shapes.

Grad [12]

Grad [Ref. 22, PR 4]

Grad [Ref. 25, Pr 4]

Grad and van Norton [21]

van Norton [17]

Burkhardt [22]

## Collisionless Shock Theory

Investigations continue on the irreversible heating of plasmas by shock-like phenomena. The situation treated concerns plasmas in which the classical collision mechanism for explaining shock structure is not applicable.

Morawetz [3] has now published a description of the internal structure of steady collisionless shocks. This theory has now been modified [30] to take into account finite electron pressures, but the problem for equal ion and electron pressures remains open.

The equations to be solved are the collisionless, steady Boltzmann equations for ions and electrons coupled with Maxwell's equations for the fields; a self-consistent solution is obtained based on asymptotic expansions in the ion-to-electron mass ratio. The electrons are treated adiabatically. These results show an irreversible transition from a constant state in front to an oscillating state behind. The case treated supposes the flow is one-dimensional; and at  $x = -\infty$ , there is no transverse magnetic field, the ion pressure is low, the Alfvén-Mach number is roughly less than 2, and the electron pressure is zero. In this analysis, the scale of length is the geometric mean Larmor radius, and in any such case the irreversible heating goes mainly to the ions.

The development of methods for solving self-consistent field problems is of much broader interest than merely for shock problems.

An important advance has been made in the analysis of strong shocks (Morton [18]), using an adiabatic two-fluid model of a collision-free plasma. Numerical computations of the transient behavior of a plasma when a perfectly conducting piston is accelerated into it, in many cases, shows that the compression waves steepen and break. This led to the discovery of the existence of steady-state generalized discontinuous solutions. Moreover, numerical analysis of the time-dependent problem revealed rapid convergence to these discontinuous solutions. Although these shocks give an irreversible heating, the relative heating of the ions and electrons can only be determined after a detailed analysis of the internal structure of those shocks is made.

In the theories described above all quantities are functions of one space variable  $x$  and time  $t$ , and the magnetic field  $\bar{B}$  is supposed to point always in the  $z$ -direction. Recent work by Saffman and Gardner has shown that there is a much wider class of solutions when this last condition is removed. In order to assess the significance of this discovery, we have begun numerical solutions of time-dependent problems with transverse magnetic fields. There are some preliminary results in the two-fluid model. For example, in the cold plasma case, with the magnetic field only in the  $z$ -direction, the compression wave is seen as a steep rise followed by an oscillating tail, whereas in some oblique field cases, there is an oscillating precursor followed by the steep rise. The feasibility



of similar computations in the Boltzmann equation case is under investigation.

Gardner and Morikawa [10] have demonstrated that the width of a small amplitude single pressure pulse, propagating normal to the magnetic field, is larger by a factor approximately  $(1 + \frac{\beta}{16} \frac{m_+}{m_-})^{1/2}$  than that in the zero temperature case. Here  $\beta$  is the ratio of gas pressure to magnetic pressure, and  $m_+ : m_-$  is the ion-to-electron mass ratio. This factor is significant unless  $\beta$  is extremely small, so that, at least for weak shocks, the zero temperature approximation is valid only for  $\beta$  very small, say  $\beta < 0.001$ .

Morawetz [3]

Morawetz [30]

Morton [18]

Gardner and Morikawa [10]

Grad and Blank [9]



## Wave Propagation

Liboff [16] has completed an analysis of the propagation of long wavelength disturbances in a plasma in a steady magnetic field, using a kinetic theory with a collision term. These results generalize previous work of Bernstein, Allis, and others. The analysis involves two linearized Boltzmann equations with modified collision terms which are coupled through the differences in temperatures and in velocities of the electron and ion gases. This analysis catalogues the infinity of modes, identifying those found in earlier more special treatments as well as discovering new modes; it also reveals interconnections between certain macroscopic and microscopic modes. Many of these were not previously known, and they only appear when the theory is as broad as this one. To cite one example from the multitude of results, there is described the intricate interplay between the fast and Alfvén modes of magneto-fluid dynamics and the kinetic plasma-magnetic modes.

There is also underway another approach using a simplified version of the Fokker-Planck equation, instead of the Boltzmann equation with collision term, which is appropriate for other parameter ranges.

Weitzner [26] has completed a study on the classical problem of the small oscillations of a plasma in a constant state without magnetic field. At time  $t = 0$  a certain disturbance is introduced at the origin  $x = 0$ , and the subsequent

wave motion is followed. In contrast with earlier results, the initial value problem is solved, giving much more information than can be obtained from plane wave solutions and dispersion relations. Quite arbitrary initial distribution functions for the plasma can be prescribed -- there are no unnatural analyticity requirements as in other treatments. Moreover, the solution can be obtained for three space dimensions. The solution has been evaluated in various asymptotic limits for large time. For stable initial distributions, one finds naturally Landau damping, and, for example, the decay of the electric field can be explicitly determined as

$$E(x,t) \sim \frac{\text{const.}}{\sqrt{t}} \cdot (\cos(\omega_p t + \frac{3\pi}{4}) + \dots).$$

For unstable initial distributions, such as for the two-stream instability, exponentially growing waves are found, and their growth rates calculated. In some cases, the actual growth rate is smaller than that which would have been predicted from the dispersion relation.

Liboff [16]

Weitzner [26]

## Other Subjects

Grad [20] presented a survey of plasma theory. This includes a greatly simplified discussion of the justification of the use of the Boltzmann equation and Fokker-Planck equation in a fully ionized plasma. In particular, it is shown that radiation coupling (cooperative fluctuation effects) play essentially no role to "dominant" order. Also included is a simplified presentation of the theory of transport coefficients and a general survey of the methods available for solution of problems.

Another survey (Grad [23]) was written on the question of the choice of practical systems of equations to be used. This includes a simplification of the theory of a guiding-center plasma;\* in particular, a description is given which makes no use of "first order drifts". There are many cases in which it is a priori clear that a guiding-center particle treatment can give no improvement over a simpler fluid analysis. Also, it is brought out that the best available theory is incapable of predicting more than rather short time plasma containment (on the basis of equilibrium theory, not stability). Whether this is an actual difficulty or a weakness in theory is not yet clear.

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\* Essentially an elaboration of H. Grad, "A Guiding Center Fluid", AEC Report TID-7503, February 1956.

Taniuti [15] investigated the effect of finite conductivity on the propagation of the magneto-fluid dynamic slow wave.

Grad [20]

Grad [23]

Taniuti [15]

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